Calibration of Oil Reservoir Simulation Codes

Maurizio Filippone

Department of Statistical Science
University College London
maurizio@stats.ucl.ac.uk

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What is the meaning of calibration?

Given a model of a physical system, calibration means:

- frequentist: estimation of model parameters
- Bayesian: inference of model parameters

Motivating application

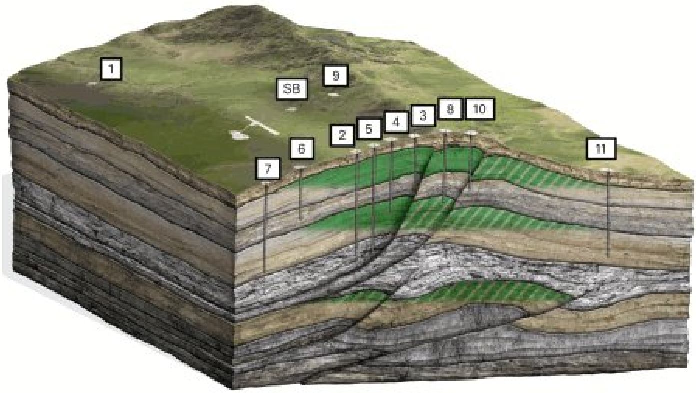
What is the problem?

- maximizing economic recovery of hydrocarbons from subsurface reservoirs

What's available?

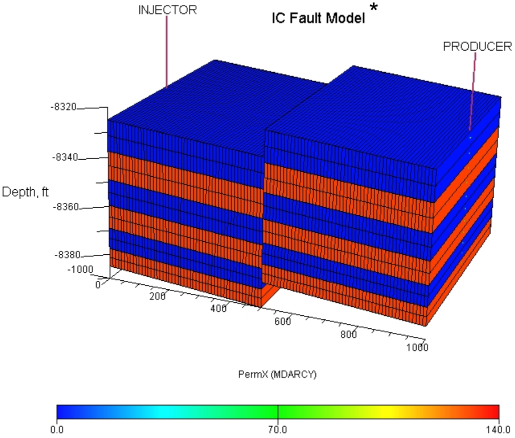
- models of the reservoirs
- geophysical data gathered on site

Example - Umiat oil field



Oil reservoir simulator

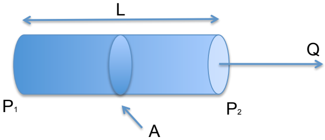
Eclipse oil reservoir simulation software by Schlumberger



Oil reservoir simulator

Reservoir simulator principles:

- conservation of energy and mass
- isothermal fluid phase behavior
- Darcy approximation (fluid flow through porous media):

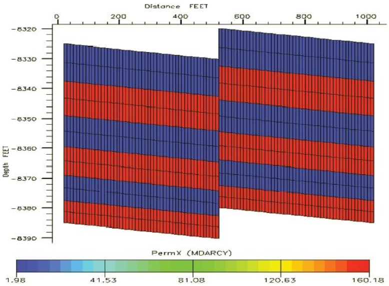


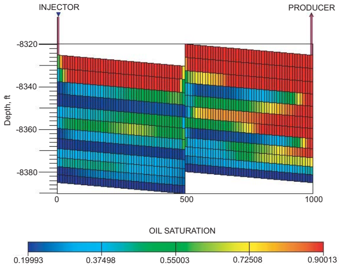
$$Q = -\frac{kA}{\mu}\frac{(P_2 - P_1)}{L}$$

- k permeability, μ viscosity
- Q discharge rate (m^3/s)

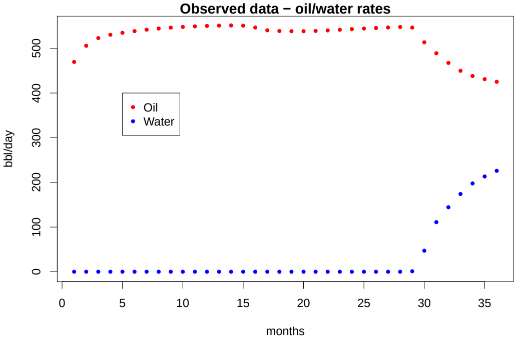
The problem - Oil reservoir model

- 2D 100 $\times$ 12 grid blocks (Tavassoli et al. 2004)
- Three parameters:
    - poor quality sand permeability (k_{low})
    - good quality sand permeability (k_{high})
    - discontinuity (throw)



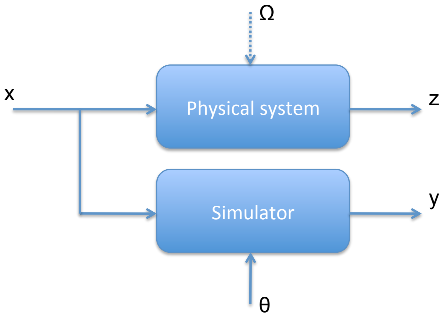


The problem - Oil reservoir model



- running time of the 2D oil reservoir model in Tavassoli et al. (2004): couple of seconds
- more complex and realistic 3D models - hours/days to run

Calibration of simulators



- We aim at inferring model parameters to quantify uncertainty in predictions

Relevance of the problem

- Calibration of simulators has application in very many application fields, e.g.:
  - high energy physics (Higdon et al. 2005)
  - geophysics (Cui et al. 2009)
  - astrophysics (Kaufman et al. 2010)
  - industrial processes (Forrester 2010)
  - ecology (Schneider et al. 2006)
  - climatology (Guillas et al. 2004, Wilkinson 2001)
  - systems biology (Wilkinson 2010)
- quantifying uncertainty is of paramount importance for balancing risks/costs of decisions

Importance of quantifying uncertainty

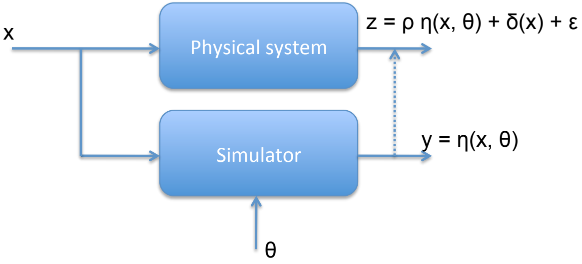
- this is what we might want
  - inferring model parameters
  - obtaining predictive distributions (balance cost of decisions)
  - Other desirables:
    - including prior information
    - approaching sequential estimation
    - doing model selection

Importance of quantifying uncertainty

- this is what we might want
  - inferring model parameters
  - obtaining predictive distributions (balance cost of decisions)
  - Other desirables:
    - including prior information
    - approaching sequential estimation
    - doing model selection
- Bayesian framework seems to be appropriate

Bayesian calibration of simulators

- Kennedy at al. (2001):

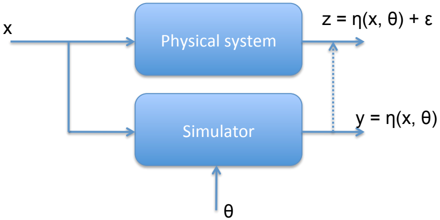


- $\eta(\mathbf{x}, \boldsymbol{\theta})$ and $\delta(\mathbf{x})$ independent and modeled using GPs
- $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ and i.i.d.

Interpretability of calibration results

- Can we really interpret the inferred simulator parameters as the “real” ones?
- if the model differs from the physical system, then the interpretation of θ is questionable
- Model misspecification and consequences in Bayesian inference - see e.g., White (1982) and Müller (2009)
- Loeppky et al. (2006) studied the problem of model misspecification in calibration problems

Simplified calibration model - Likelihood



- we have direct access to the log-likelihood:

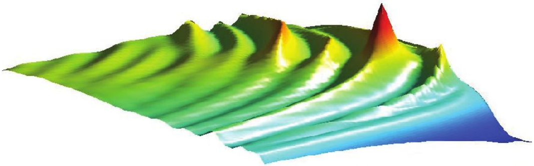
$$\log[p(\mathrm{Data}|\boldsymbol{\theta})] = \sum_i \log\left[\mathcal{N}(z_i|\eta(\mathbf{x}_i, \boldsymbol{\theta}_i), \sigma_\varepsilon^2)\right]$$

- if we place priors $\pi(\boldsymbol{\theta})$ we can infer $\boldsymbol{\theta}$ - simulation using Markov chain Monte Carlo (MCMC)

$$p(\boldsymbol{\theta}|\mathrm{Data}) \propto p(\mathrm{Data}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})$$

Posterior inference - Multimodalities

- Given that the simulator is usually modeling a complex physical system, the likelihood is often multimodal

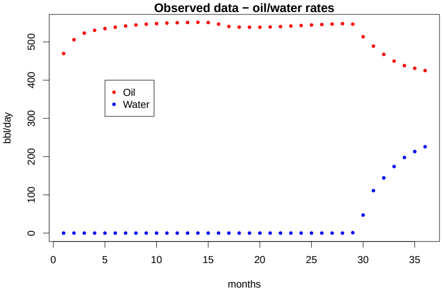


Oil reservoir data

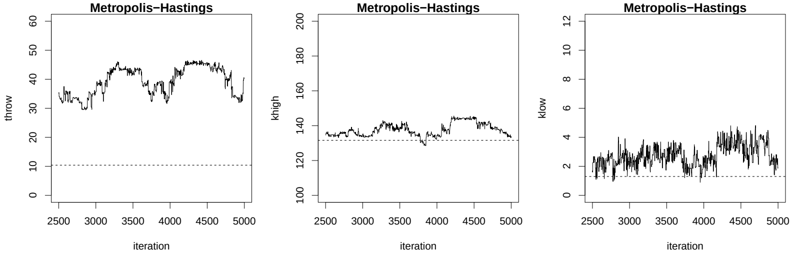
- As a working example let us consider the 2D oil reservoir model by Tavassoli et al. (2004)
- "Real" data are generated from the simulator:

$$z_i = \eta(\mathbf{x}_i, \tilde{\theta}) + \varepsilon_i$$

where $\tilde{\theta}$ represents a set of "true" parameters



Inference using Metropolis-Hastings



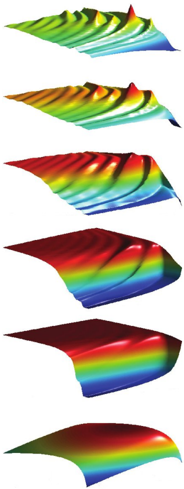
- Poor exploration of the parameter space
- Proper exploration of the parameter space via population based Markov chain Monte Carlo (Pop-MCMC)

Inference using Pop-MCMC

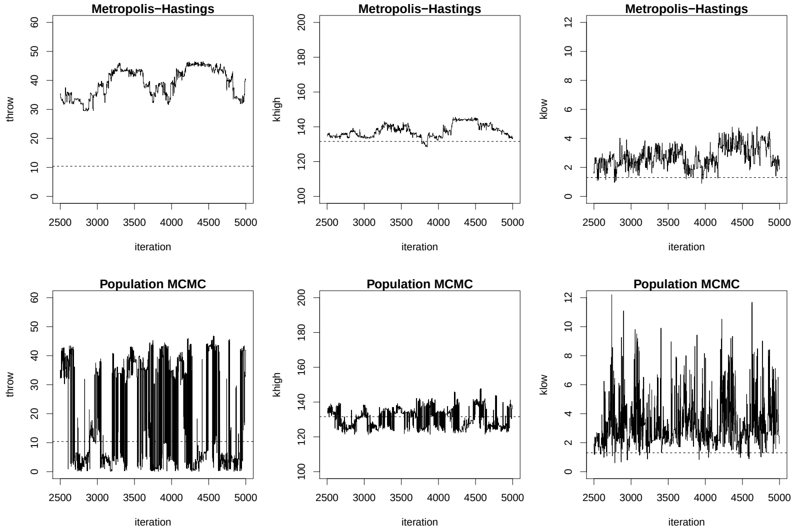
- bridge from the prior to the posterior via tempering

$$\text{tempered posterior} \propto p(\mathrm{Data}|\boldsymbol{\theta})^t \pi(\boldsymbol{\theta})$$

  with $t \in [0, 1]$
- one sampler for each tempered posterior
- samplers sampling independently and exchanging samples so that invariance of $p(\boldsymbol{\theta}|\mathrm{Data})$ is preserved

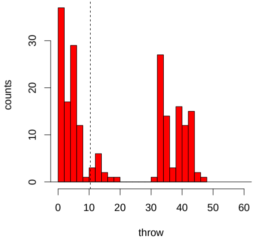


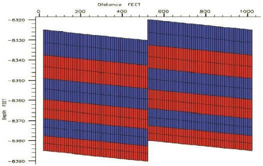
Inference using Pop-MCMC

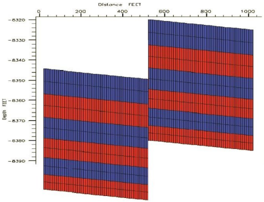


Inference using Pop-MCMC - Results

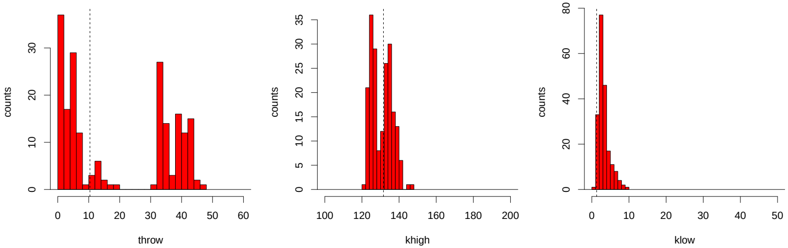
Interpretation of the multimodal posterior - throw parameter

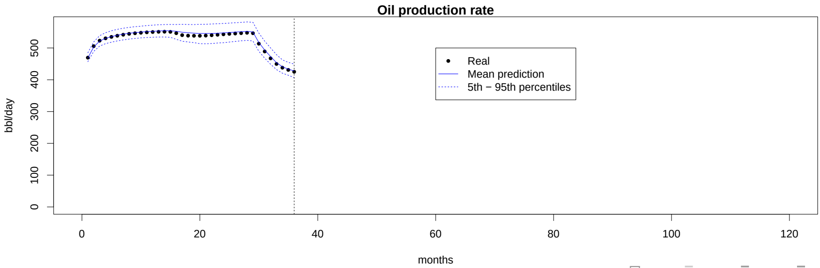






Inference using Pop-MCMC - Results





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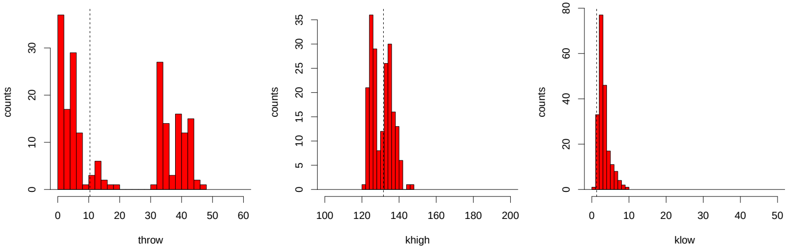

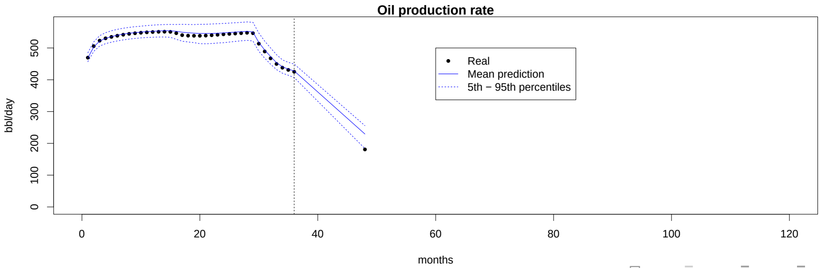

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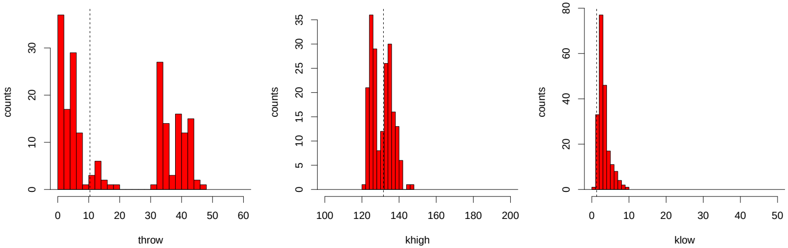
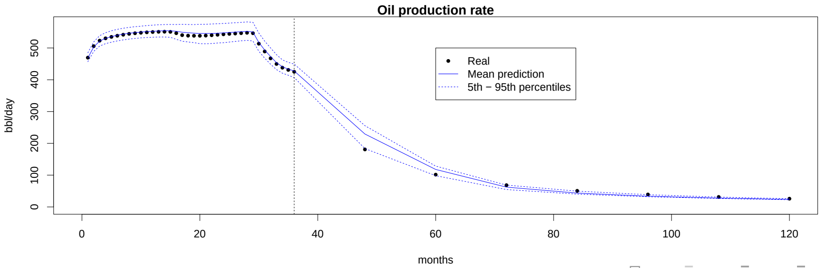


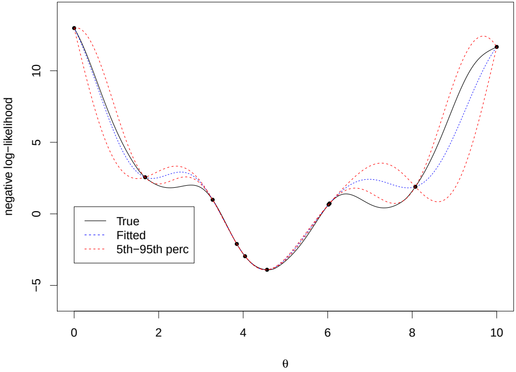


What if the running time of the simulator is prohibitive?

- Simulators could be so complex to require hours or even days to run
- Fully Bayesian treatment is not viable
- We aim at using the available computational resources to find parameters and an estimate of the uncertainty (not Bayesian)

Need for emulators

- Emulators as a proxy for the expensive likelihood
- Start from a set of design points (latin hypercubes)
- Emulator using Gaussian Process



Need for emulators

- Marginals of the emulator are Gaussian:

$$y(\boldsymbol{\theta}) \simeq \mathcal{N}(\mu(\boldsymbol{\theta}), s^2(\boldsymbol{\theta}))$$

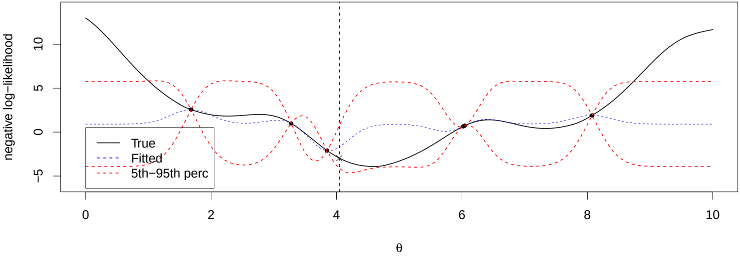
- Incremental design for minimization optimizing a utility (improvement) function (Jones et al. 1998)

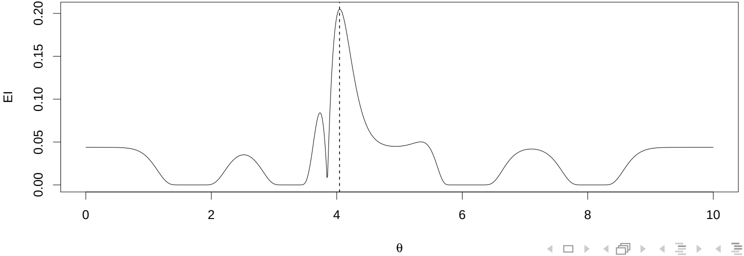
$$I(\boldsymbol{\theta}) = \max(f_{\text{min}} - y(\boldsymbol{\theta}), 0)$$

expected value:

$$\mathrm{E}[I(\boldsymbol{\theta})] = (f_{\text{min}} - \mu(\boldsymbol{\theta}))\Phi\left(\frac{f_{\text{min}} - \mu(\boldsymbol{\theta})}{s(\boldsymbol{\theta})}\right) + s(\boldsymbol{\theta})\mathcal{N}\left(\frac{f_{\text{min}} - \mu(\boldsymbol{\theta})}{s(\boldsymbol{\theta})}\right)$$

Example of incremental design





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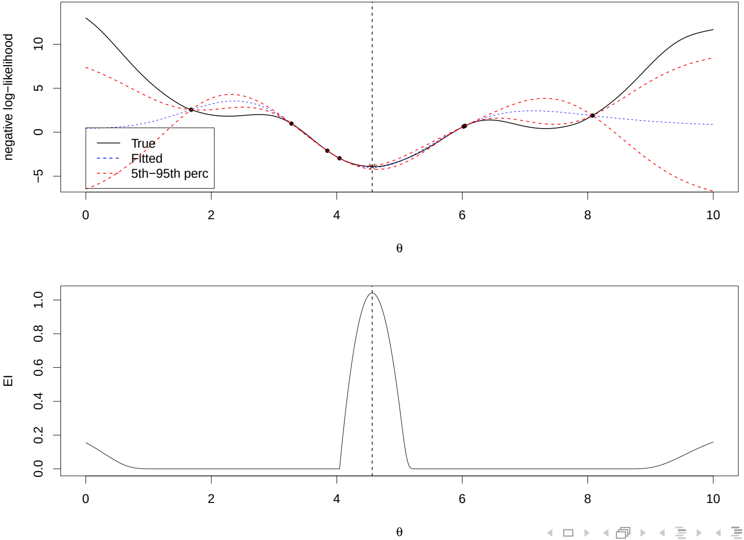

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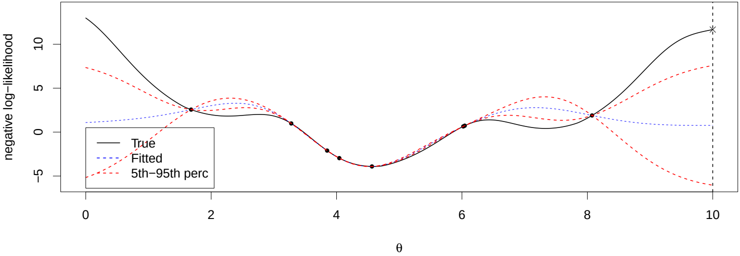

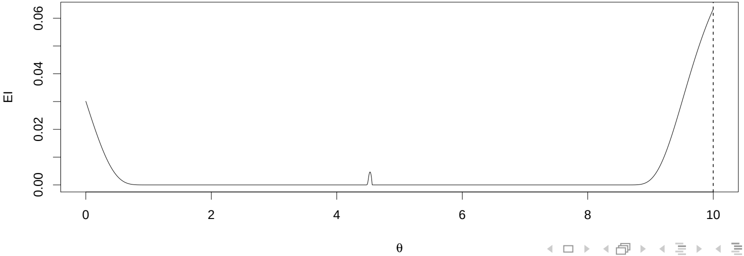

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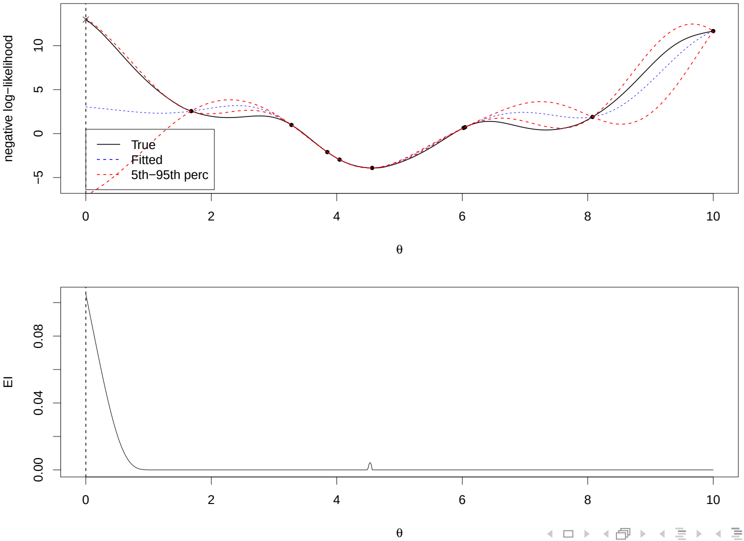

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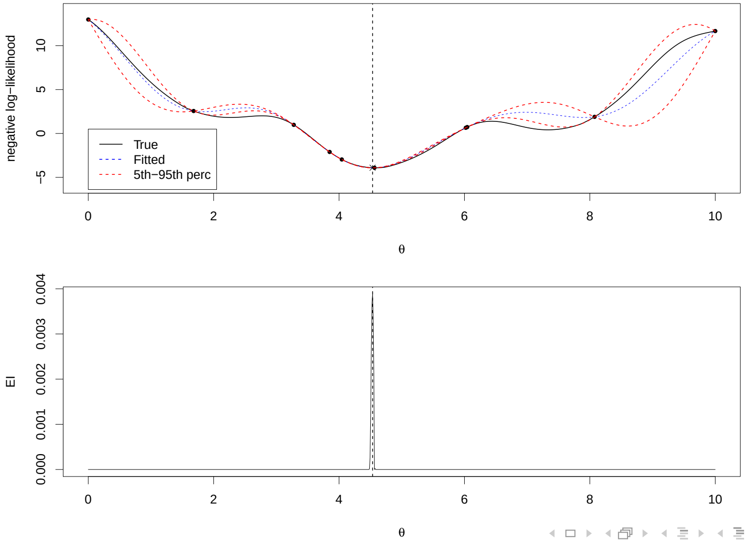

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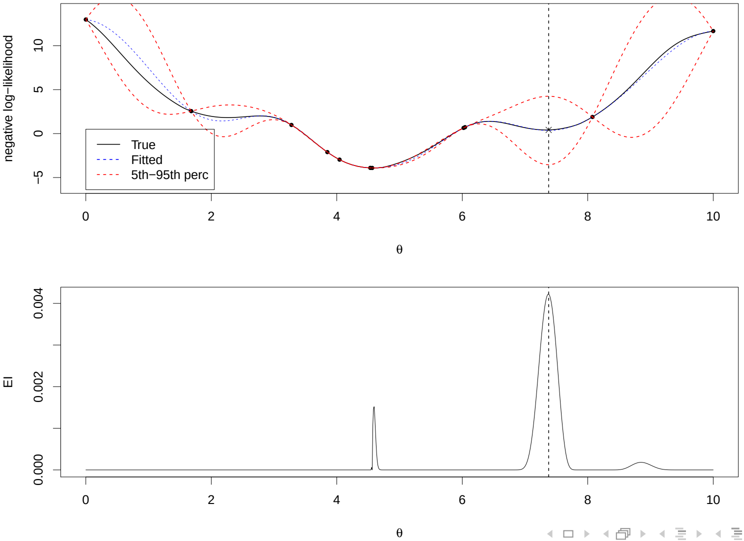

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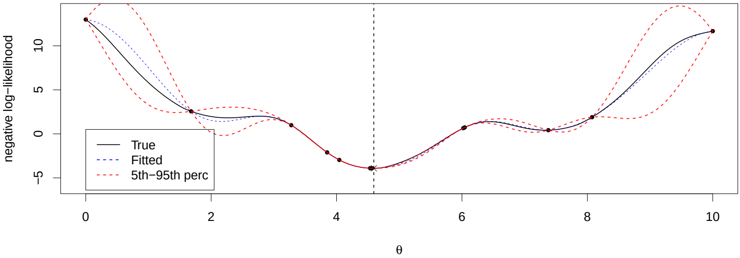

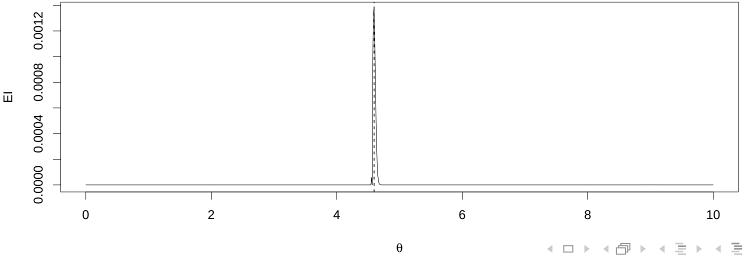

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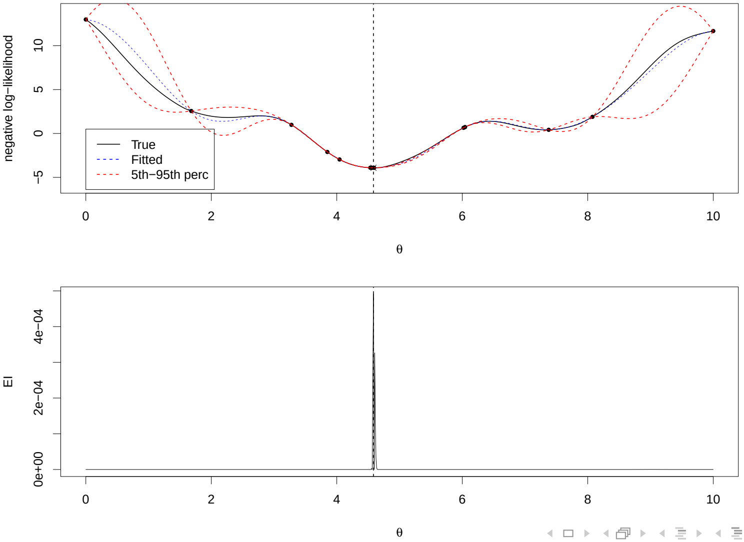

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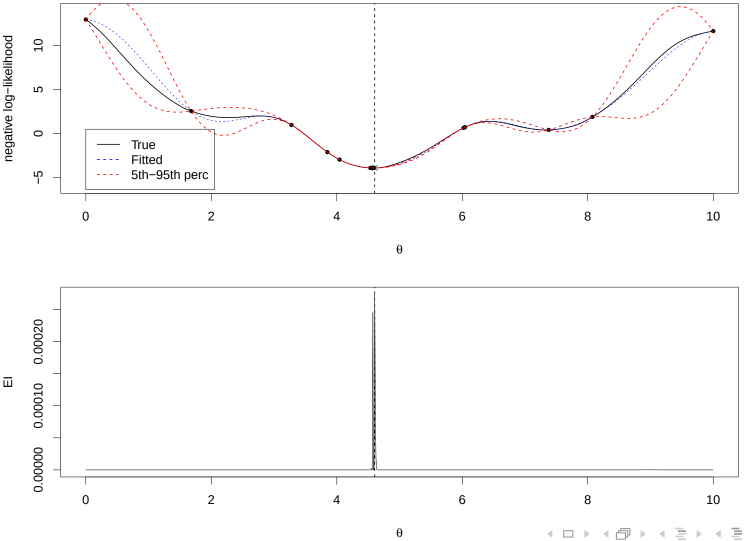
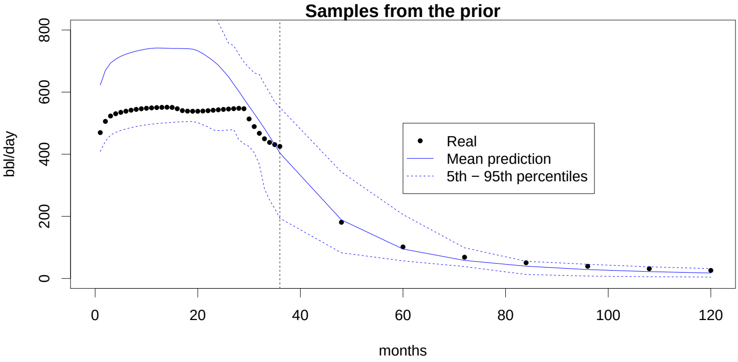


Incremental design - Oil reservoir - Results

- incremental design on the oil data
- we assumed that computational resources allowed only 100 runs of the simulator
  - 50 simulations using latin hypercubes
  - 50 simulations using incremental design

Incremental design - Oil reservoir - Results

true values: 10.4, 131.6, 1.3



Incremental design - Oil reservoir - Results

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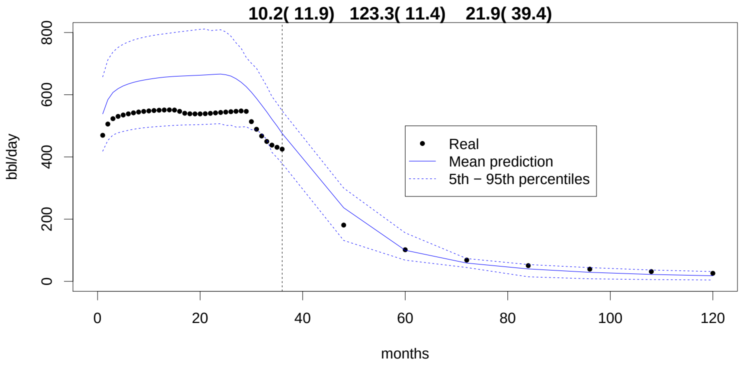

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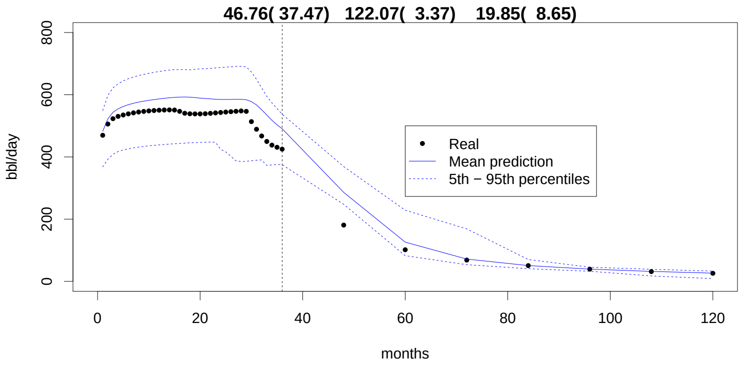

Conclusions and ongoing work

- This work is part of the ongoing research program of the Computational Statistics group at UCL on inference in complex systems
- Calibration of simulators is an important and promising research area
- It is a challenging problem even with simplifying assumptions:
  - inference over parameters of complex simulators
  - expensive to evaluate the likelihood

Conclusions and ongoing work

Ongoing research:

- Composite likelihoods
- Emulators for moderate/large sized data sets
- Consistency of estimators in incremental experiment design?
- Hybrid/Manifold Monte Carlo with emulator as potential field
- Design of covariance functions for emulators of differential equations based simulators
- Model selection

Acknowledgements

[1] L. Mohamed, B. Calderhead, M.Filippone, M. Christie, M. Girolami.
Population MCMC methods for history matching and uncertainty quantification.
Computational Geosciences. to appear

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